MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EME3156-COMPUTATIONAL METHODS FOR MECHANICAL ENGINEERING

(ME)

8 MARCH 2019

3.00 P.M.– 5.00 P.M.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 9 pages including the cover page.
2. This paper is divided into **Sections A , B and C**. Questions in **Section A** are **compulsory**. Choose either (a) or (b) to answer in Section B. Choose **TWO** questions to answer in **Section C**.
3. The distribution of the marks for each question is given.
4. Please write all your answers in the answer booklet provided.
5. All necessary workings **MUST** be shown.

Section A

Question 1

Equation (1.1) shows a first order ODE with initial condition,

$$\frac{dy}{dx} = y - 2x \ , \quad y(0) = 1 \tag{1.1}$$

a) By using midpoint method with step size $h = 0.15$, find $y(0.3)$. [5 marks]

b) By using fourth order Runge-Kutta (RK4) with step size $h = 0.3$, find $y(0.3)$. [8 marks]

c) Determine the exact value of $y(0.3)$, given that the exact solution is $y(x) = 2x + 2 - e^x$. Which method above, (a) or (b), is more accurate in finding $y(0.3)$? [2 marks]

Question 2

Consider a nonlinear system below:

$$3 - x^2 - y = 0$$

$$x + 1 - y = 0 \tag{2.1}$$

a) Compute the first iteration by using steepest descent method, starting from $x^{(0)} = (0,0)^T$. [11 marks]

b) Explain the steps involved, if the above problem is to be solved by using Newton method. [4 marks]

Continued...

Section B

Question 3

Equation (3.1) shows a second order ODE with the boundary conditions. The domain is discretized into five nodes equally distanced.

$$\frac{d^2y}{dx^2} = 0.02y + 1, \quad y(0) = 30, \quad y(10) = 120 \qquad (3.1)$$

a-i) Sketch the discretization with the node numbering in sequence starting from "1" at the left boundary.

[2 marks]

a-ii) Write down the finite difference form of Equation (3.1) by using the central difference for all terms, and then apply this equation to the discretized domain. The boundary conditions must be incorporated into these equations. Rearrange the difference equations into matrix form $[K]\{y\} = \{F\}$.

[11 marks]

a-iii) Use Gaussian Elimination method to solve the matrix equation.

[7 marks]

b-i) Equation (3.1) can also be solved by using shooting method. Write down two first order ODEs with one assumption of initial condition. Set the initial condition to be of zero value. Introduce two new variables to the problem, i.e. "U" and "V". [3 marks]

b-ii) Use explicit Euler method with step size h of 2.5 to find y(10). [12 marks]

b-iii) It is unlikely that your first trial will match the final solution, i.e. $y(10)= 120$. Explain your next step to overcome the problem. Explain extra steps one need to take if the above problem is nonlinear in nature. [5 marks]

Continued...

Section C

Question 4

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{D(x,y)}{K}=0 \tag{4.1}$$

Equation (4.1) describes the steady-state temperature distribution u on a heated metal plate which also satisfies the Poisson equation. $D(x,y)$ is the power density at (x,y) and K is the thermal conductivity. The power is generated at a constant rate of $D(x,y) = 5$ watts/cm^3. The thermal conductivity of the metal plate is $K = 3.85$ watts/cm°C. The boundary values are all at 25°C.

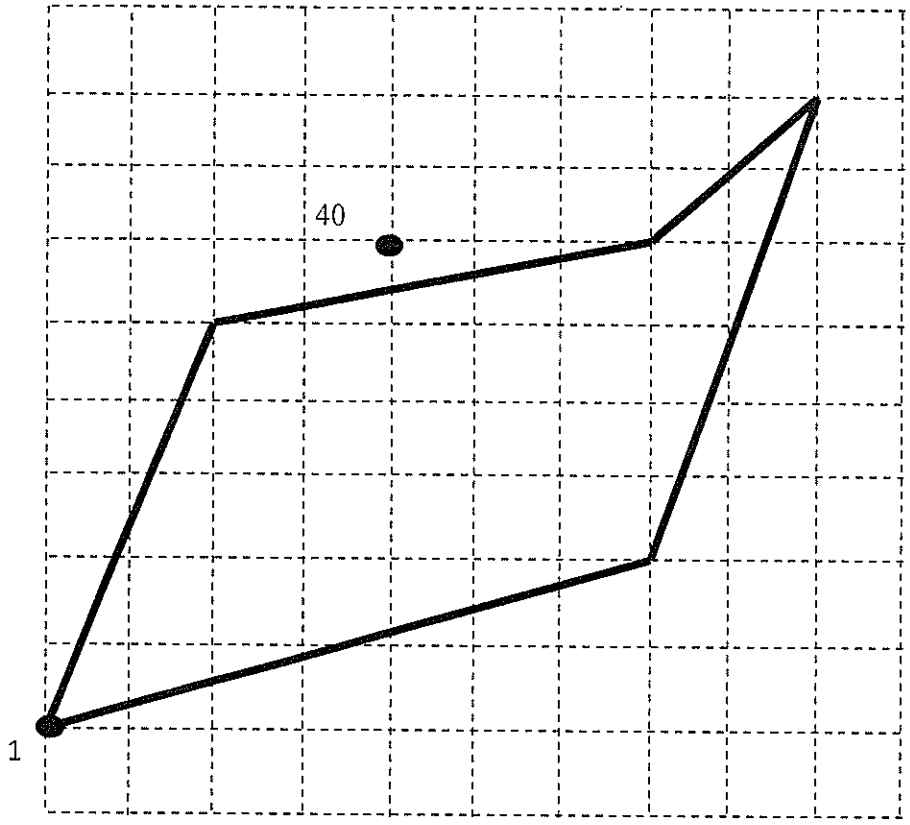

Figure 1: Irregular shape in a gridded rectangular box

Continued...

For the irregular shape in Fig. 1 above, each small rectangle box is of the dimension 2cm x 2cm. Node 1 is considered as the origin of the x-y coordinate system and it is labeled in Fig.1. Node 40 is also shown in the figure. Copy the figure into your answer sheet and label all the missing nodes. The labelling of the nodes start from left to right and then goes from bottom to top.

a) Discretise the domain. By using central difference of the second derivative, find the finite difference form of the equation (4.1).

[4 marks]

b) Write down all the nodes to be solved. [4 marks]

c) Write down the difference form of equation (4.1) for nodes 12 and 20. Boundary values are to be included in the equations. [4 marks]

d) Rewrite equation (4.1) in matrix form representing the iteration

$$[A]\{u\} = \{c\} \qquad (4.2)$$

where $\{u\}$ is a vector for nodal variable at a two-dimensional location and $\{c\}$ is the vector after including water table heights at the boundaries. $[A]$ is a square matrix.

[13 marks]

Continued...

Question 5

The governing equation of a one-dimensional, transient heat conduction problem is given in Equation (5.1). The boundary and initial conditions are given in Equation (5.2). The domain is discretised into 6 nodes equally distanced.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + Q, \ \alpha = 1, Q = 10 \qquad (5.1)$$

$$\frac{\partial T(0,t)}{\partial x} = 2, T(1,t) = 100, T(x,0) = 100x \qquad (5.2)$$

a) Sketch the discretisation with the node numbering in sequence starting from "1" at the left boundary. [2 marks]

b) By using the explicit Euler method with $\Delta t = 0.01$, write down the finite difference from the Equation (5.1). Apply this equation to the discretised domain.

[3 marks]

c) State clearly which equation in (5.2) represents the boundary and initial conditions respectively. Discretize the conditions. [6 marks]

d) Assemble the equations in (b) to become

$$\{T\}^t = [A]\{T\}^{t+1} + \{Boundary\ Conditions\}.$$

$\{T\}^{t+1}$ and $\{T\}^t$ are vectors for nodal variables at time t+1 and t respectively.

$[A]$ is a square matrix. [5 marks]

e) Solve for vector $\{T\}^1$ [4 marks]

f) If the equation (5.1) is to be solved using Implicit method, write down the difference equation to be used. [5 marks]

Continued...

Question 6:

The wave equation of a vibrating spring is given by

$$16\frac{\partial^2 u}{\partial t^2} - 9\frac{\partial^2 u}{\partial x^2} = 0\ ,\ 0 < x < 2,\ t > 0 \qquad (6.1)$$

a) By using central difference of the second derivative, approximate the derivative $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial t^2}$ in their respective variables. Thus write out equation (6.1) in finite difference form. [3 marks]

b) Given that

$$\frac{\partial u}{\partial t}(x,0) = \pi\, sin\left(\frac{\pi}{3}x\right),\quad u(x,0) = \sin(\pi x)$$

Both ends of the spring are fixed to the walls. The spatial domain is also discretised into 6 nodes equally spaced. Use $\Delta t = 0.4$ for your calculation.

Rewrite equation (6.1) in matrix form,

$$\{u\}^{t+1} = [A]\{u\}^{t} + \{u\}^{t-1} + \{BC\}$$

where $\{u\}^{t+1}$, $\{u\}^{t}$ and $\{u\}^{t-1}$ are vectors for nodal variables at time t+1, t and t-1 respectively. $[A]$ is a square matrix while $\{BC\}$ are boundary conditions.

[16 marks]

c) Evaluate $\{u\}^{0.4}$. [6 marks]

Continued...

Appendix

Explicit Euler method

$$y_{i+1} = y_i + h\, f(x_i, y_i)$$

Implicit Euler method

$$y_{i+1} = y_i + h\, f(x_{i+1}, y_{i+1})$$

Modified Euler method (predictor-corrector)

Step 1: $K_1 = f(x_i, y_i),\ y_{i+1}^P = y_i + hK_1$

Step 2: $K_2 = f(x_{i+1}, y_{i+1}^P),\ y_{i+1}^C = y_i + h\,\frac{1}{2}(K_1 + K_2)$

Midpoint method

Step 1: $K_1 = f(x_i, y_i),\ y_m = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_m),\ y_{i+1} = y_i + hK_2$

Fourth-order Runge-Kutta

Step 1: $K_1 = f(x_i, y_i),\ y_{m1} = y_i + \frac{h}{2}K_1$

Step 2: $K_2 = f(x_m, y_{m1}),\ y_{m2} = y_i + \frac{h}{2}K_2$

Step 3: $K_3 = f(x_m, y_{m2}),\ y_{m3} = y_i + hK_3$

Step 4: $K_4 = f(x_{i+1}, y_{m3}),\ y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

Second-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

Third-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{12}(23f_i - 16f_{i-1} + 5f_{i-2})$$

Fourth-order Adams-Bashforth

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3})$$

Second-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{2}(f_i + f_{i+1})$$

Third-order Adams-Moulton

$$y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

Continued...

Fourth-order Adams-Moulton

$$y_{i+1} = y_i + \tfrac{h}{24}\left(9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2}\right)$$

First derivative, forward difference

$$y' = \tfrac{1}{h}\left(y_{i+1} - y_i\right)$$

First derivative, backward difference

$$y' = \tfrac{1}{h}\left(y_i - y_{i-1}\right)$$

First derivative, central difference

$$y' = \tfrac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

Second derivative, forward difference

$$y'' = \tfrac{1}{h^2}\left(y_i - 2y_{i+1} + y_{i+2}\right)$$

Second derivative, backward difference

$$y'' = \tfrac{1}{h^2}\left(y_{i-2} - 2y_{i-1} + y_i\right)$$

Second derivative, central difference

$$y'' = \tfrac{1}{h^2}\left(y_{i-1} - 2y_i + y_{i+1}\right)$$

Jacobi method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Gauss-Seidel method

$$x_i^{k+1} = \frac{1}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Successive over-relaxation method

$$x_i^{k+1} = (1-\omega)x_i^k + \frac{\omega}{a_{ii}}\left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij}x_j^k\right)$$

Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial f_i}{\partial x_1} & \frac{\partial f_i}{\partial x_2} & \cdots & \frac{\partial f_i}{\partial x_n} \\ \frac{\partial g_i}{\partial x_1} & \frac{\partial g_i}{\partial x_2} & \cdots & \frac{\partial g_i}{\partial x_n} \\ \frac{\partial h_i}{\partial x_1} & \frac{\partial h_i}{\partial x_2} & \cdots & \frac{\partial h_i}{\partial x_n} \\ \vdots & \vdots & \cdots & \vdots \end{bmatrix}$$

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